used, there will be a time lag in transferring the instrument to the opposite bank during which time the value of refraction may change. Therefore, to ensure better results, some surveyors recommend the use of two levels, one at each bank, so that sights are taken simultaneously. Although this will give better results but each level may have a different collimation error. The instruments should therefore be interchanged and the entire procedure repeated. The mean of the four values will be the most probable difference in the level between the two points.

Example 6.18 The following notes refer to the reciprocal levels taken with one level:

Instrument station	Staff readings on A	Staff readings on B	Remarks
A	1.03	1.630	Distance AB = 800 m
B	0.95	1.540	R.L. of A = 450 m

Find:

(i) true R.L. of B

(ii) combined correction for curvature and refraction

(iii) the error in collimation adjustment of the instrument.

Solution

(i) *True R.L. of B*

Instrument at A

Incorrect level difference between A and B = 1.630 − 1.03 = 0.600 m

Instrument at B

Incorrect level difference between A and B = 1.540 − 0.95 = 0.59 m

True difference of level between A and B = mean of the two incorrect differences

$$= \frac{0.6 + 0.59}{2}$$

$$= 0.595 \text{ m} \quad \text{(fall from A to B)}$$

The results can also be obtained by using the expression

$$h = \frac{(b - a) + (c - d)}{2} = \frac{(1.630 - 1.03) + (1.540 - 0.95)}{2} = 0.595 \text{ m}$$

(ii) *Combined correction for curvature and refraction*

$$= 0.0673\ D^2$$

$$= 0.0673 \left(\frac{800}{1000}\right)^2 = 0.043 \text{ m}$$

(iii) *Error in collimation adjustment*

Reading of A = 1.03 m

Fall from A to B = 0.595 m

Required reading of level lime = 1.03 + 0.595 = 1.625 m

The actual staff reading at B (touching horizontal line)

= 1.625 + 0.043 = 1.668 m

But the observed reading at B = 1.630 m

Error in collimation adjustment = 1.668 − 1.630 = 0.038 m

Error of collimation is negative since the observed reading is less than the actual.

PRECISE LEVELLING 6.20

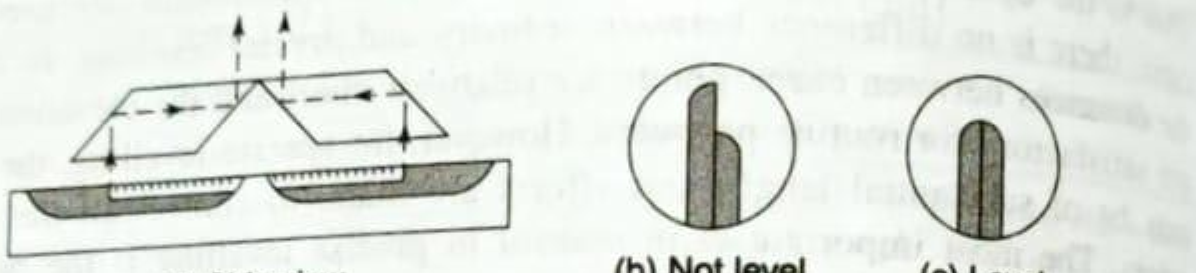
This is the operation of levelling in which precise instruments are used. In principle, there is no difference between ordinary and *precise levelling*. In the former, the distances between check points are relatively short and the elevations obtained are satisfactory for routine purposes. However, for precise levelling, the level loop may be of substantial length and efforts are made to control all the sources of errors. The most important error control in precise levelling is the balancing of foresight and backsight distances. The lengths of sights are kept equal to within 0.5 m. This has the effect of eliminating the residual collimation error and errors due to curvature, and minimizing errors due to refraction. These distances can be readily computed if the instrument is fitted with stadia wires.

Temperatures are read at intervals to correct the graduations along the length of the staff. some of the other effects of change of temperature are: the coefficient of refraction is altered, shimmering is caused, and unequal expansion and contraction of the instrument is there. The effect of change of coefficient of refraction is probably cancelled by equalizing backsight and foresight distances. Shimmering can be minimized by avoiding observations near the foot of the staff. The unequal expansion or contraction of the instrument can be guarded against by keeping the instrument under shade. To minimize errors of reading, sights should be kept short at about 50 m, and 98 m should be regarded as the maximum distance. Precise levelling is used for establishing benchmarks with high precision by some government agencies such *as Great Trigonometrical Survey of India Department.*

In principle, the instruments used are the same as those used for ordinary levelling but special attention is paid by the manufacturers to some of the construction details adding to the precision. Levels manufactured by Carl Zeiss, Wild Heerburg and Nikon Corporation are commonly used for precise levelling. These levels differ from the ordinary levels in the following respects;

1. The levelling base is made broad and the part of the instrument above the base is reduced in height. This increases stability.
2. The radius of the bubble tube curvature is made large to increase its sensitivity. The value of 2 mm division is usually 1.2 − 3 seconds.
3. The aperture of the telescope should be at least 4 cm and the focal length about 37.5 − 50 cm. Magnification of telescope ranges from about 30 to over 50 diameters.
4. The bubble is capable of being read at the eye end and in some models in the main telescope itself.
5. An automatic or a tilting level is employed. When a tilting level is used, the final adjustment for the level in the direction of the line of sight is made with the tilting screw. A parallel plate micrometer is fitted, making it possible to read a staff directly to 0.001 m.

6. The level is provided with a *coincidence bubble tube*. It facilitates two ends of the bubble to be seen side by side through an auxillary eyepiece (Fig. 6.41 (a)), fitted below the eyepiece end of the telescope. This is achieved with the help of an optical arrangement which splits the bubble image into two parts and makes them to appear adjacent to each other as shown in Fig. 6.41 (b). When the bubble is centred the two ends appear to coincide and form a U-shaped curve as shown in Fig. 6.41 (c).



(a) Side view (b) Not level (c) Level

Fig. 6.41 *Working of coincidence bubble tube*

The level page for precise levelling is somewhat different than that of ordinary levelling and is as shown in level book page 3 (Table 6.4). There is no need to compute each H.I. and elevation of the turning points, since this is resorted to only for establishing a B.M. The desired difference in elevation between benchmarks is merely the difference between sum of the backsight and foresight readings. To equalize backsight and foresight distances, stadia wires are also read in addition to the horizontal cross-wire as shown in the level page book 3. Though costly and time consuming, the accuracy of the staff reading is considerably improved by reading all the three wires and taking the mean of the three readings.

Table 6.4 *Level book page 3*

Station	*B.S.*	*H.I.*	*F.S.*	*R.L.*	*Temperature (°C)*	*Remarks*
	1.530					
1	2.100					B.M.
	2.670					
Mean	2.100	128.425		126.325		
	1.260		1.200			
2	1.900		1.831			T.P.
	2.541		2.463			
Mean	1.900		1.831	126.594		
			1.382			
3		128.494	1.964			
			2.645			
		Mean	1.964	126.530		
	Σ B.S. − Σ F.S. = 4.000 − 3.795 = 0.205			Last R.L. − First R.L. = 126.530 − 126.325 = 0.205		

Thus, precise levelling involves the use of suitable instruments, protection of the level from disturbing elements, careful manipulation of the level, and a program of procedures designed to reduce the errors.

Standards for Permissible Error in Precise Levelling

First order	$\pm 4\sqrt{k}$ mm
Second order	$\pm 8\sqrt{k}$ mm
Third order	$\pm 12\sqrt{k}$ mm
For a closed loop	$\pm 24\sqrt{k}$ mm

Here k is distance in km.

FLY LEVELLING 6.21

It is an operation of levelling in which a line of levels is run to determine the approximate elevations. it is carried out for reconnaissance of the area.

Example 6.19 In running fly levels from a B.M. of R.L. 250.00 m, the following readings (in m) were obtained:

Backsight:	1.315,	2.035,	1.980,	2.625
Foresight:	1.150,	3.450,	2.255	

From the last position of the instrument, five pegs at 20 m interval are to be set out on a uniform rising gradient of 1 in 40. The first peg is to have a R.L. of 247.245 m. Work out the staff readings required for setting the tops of the pegs on the given gradient.

Solution Enter the B.S. and F.S. readings in the level book page and work out the reduced levels of stations.

Station	*B.S.*	*I.S.*	*F.S.*	*H.I.*	*R.L.*	*Remarks*
1	1.315			251.315	250.000	B.M.
2	2.035		1.150	252.200	250.165	C.P.
3	1.980		3.450	250.730	248.750	C.P.
4	2.625		2.255	251.100	248.475	C.P.
5		3.855			247.245	Peg 1
6		3.355			247.745	Peg 2
7		2.855			248.245	Peg 3
8		2.355			248.745	Peg 4
9			1.855		249.245	Peg 5
Check	Σ B.S. − Σ F.S. = 7.955 − 8.710 = − 0.755			Last R.L. − First R.L. = 249.245 − 250 = − 0.755		

The first peg is fixed with the R.L. of its top at 247.245 m and the height of instrument is 251.150 m. The R.L. of the subsequent pegs at 20 m interval will depend upon the rising gradient of 1 in 40. Difference in level, between two consecutive readings = distance/gradient = 20/40 = 0.5 m. The ground is rising by 0.5 m between the consecutive pegs.

(iv) *Adjustment*

(1) Bring the bubble halfway back by foot screws and half by raising or lowering one wye relative to the other by means of screws which join the base of the wye to the stage.

(2) Repeat the test and adjustment till correct.

CASE B

(i) Adjustment of Line of Sight : Same as for case A.

(ii) Adjustment for the Perpendicularity of the Vertical Axis and Level Tube

The test is done in the same way as adjustment *(iii)* for *case A*, but the error is adjusted half by means of foot screws and half by means of capstan screws of the bubble tube.

(iii) Adjustment for Parallelism of Line of Sight to the Axis of the Level Tube

(i) *Test*

(1) Level the instrument carefully by keeping the telescope parallel to two foot screws. Clamp the motion about vertical axis.

(2) Keep a level rod in the line of sight and take the reading.

(3) Reverse the telescope end for end in the wyes and again sight the staff.

(4) If the reading is the same, the instrument is in adjustment. If not, it requires adjustment.

(ii) *Adjustment*

Bring the line of sight to the mean reading on the staff by means of adjusting screws under one wye.

17

Precise Levelling

17.1. INTRODUCTION

Precise levelling is used for establishing bench marks with great accuracy at widely distributed points. The precise levelling differs from the ordinary levelling in the following points :

(i) High grade levels and stadia rods are used in precise levelling.

(ii) Length of sight is limited to 100 m in length.

(iii) Rod readings are taken against the three horizontal hairs of the diaphragm.

(iv) Backsight and foresight distances are precisely kept equal, the distances being calculated from stadia hair readings.

(v) Two rodmen are employed and backsight and foresight are taken in quick succession.

(vi) The adjustments of the *precise level* are tested *daily* and the correction applied to the rod readings. The rod is standardized frequently.

The precise levelling can be classified under the following three heads, depending upon the permissible errors :

First order : permissible error = 4 mm $\sqrt{K}$ or 0.017 ft $\sqrt{M}$

Second order : permissible error = 8.4 mm $\sqrt{K}$ or 0.035 ft $\sqrt{M}$

Third order : permissible error = 12 mm $\sqrt{K}$ or 0.05 ft $\sqrt{M}$.

For most of the engineering surveys, permissible error of closure of a level circuit is 0.1 $\sqrt{M}$ or 24 mm $\sqrt{K}$. The construction engineer, therefore, is accustomed to refer to any of the three higher orders as precise levelling.

17.2. THE PRECISE LEVEL

The precise levelling instrument has, generally, a telescope of greater magnifying power (40 to 50 D). It is provided with three parallel plate screws and a very sensitive bubble which is brought to the centre for each reading by a fine tilting drum placed under the eyepiece. Thus, the line of sight can be made horizontal even when the instrument as a whole is not exactly level.

The bubble can be seen from the eyepiece end of the telescope by reflection in the small prism above the bubble tube. Coincidence system is used for centring the bubble, as shown in Fig. 17.1. An adjustable mirror placed immediately below the bubble tube illuminates the bubble. One-half of each extremity of the bubble is reflected by the prism in the long rectangular casing immediately above the bubble tube into the small prism box. When the bubble is not perfectly central, the reflections of the two halves appear as shown in Fig. 17.1 (*a*). When the bubble is central, the reflection of the two halves makes one curve as shown in Fig. 17.1(*b*). The bubble tube generally has sensitiveness of 10 seconds of arc per 2 mm graduation.

Left : not in coincidence, right : coincidence

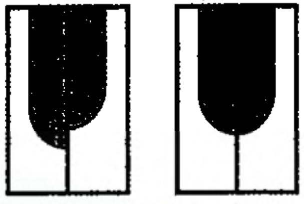


FIG. 17.1

17.3. WILD N-3 PRECISION LEVEL

Fig. 17.2 shows the photograph of Wild N-3 precision level for geodetic levelling of highest precision, construction of bridges, measurements of deformation and deflection, determination of the sinking of dams, mounting of large machinery etc. Apart from the main telescope, the level contains two optical micrometers placed to the left of the eyepiece—one is meant for viewing the coincidence level and the other is for taking the micrometer reading (Both the auxiliary telescopes are not visible in the photograph since right-hand view has been shown). The tilting screw (2) has fine pitch and is placed below the eyepiece and for fine movement in azimuth, it also contains a horizontal tangent screw (4). The micrometer knob (6) is used for bringing the image of the particular staff division line accurately between the V-line of the graticule plate.

The centring of the bubble is done by means of prism-system in which the bubble-ends are brought to coincidence (Fig. 17.1). The optical micrometer is used for reading the staff. Fig. 17.3 shows the field of view through all three eyepieces. The graticule has a horizontal hair to the right half and has two inclined hairs, forming V-shape, to the left hair. After having focused the objective, the approximate reading of the staff may be seen. The optical micrometer is used for fine reading of staff. By turning the knob (6) for micrometer, the plane parallel glass plate mounted in front of the objective is tilted and the image of the particular staff division line is thus brought accurately between the V-lines of the graticule plate. This displacement of the line of sight, to a maximum of 10 mm, is read on a bright scale in the measuring eyepiece to $\frac{1}{100}$ mm. Thus, the staff reading (Fig. 17.3) is $148 + 0.653 = 148.653$ cm. An invar rod (Fig. 17.6) is used with this level. The manufacturers claim an accuracy of ± 0.001 inch in a mile of single levelling.

17.4. THE COOKE S-550 PRECISE LEVEL

Fig. 17.4 shows the photograph of the Cooke S-550 precise level manufactured by M/s Vickers Instruments Ltd. used for geodetic levelling, determination of dam settlement and ground subsidence, machinery installation, and large scale meteorology. The telescope spirit vial is illuminated by a light diffusing window. The vial is read through the telescope eyepiece by an optical coincidence system. The telescope is fitted with a calibrated fine levelling screw, one revolution tilting the telescope through a vertical angle corresponding to 1 : 1000. The micrometer head is sub-divided into fifty parts, one division, therefore, being equal to 1 in 50,000. The extent of calibration is twenty revolutions, corresponding to an angle of 1 in 50. The reticule has vertical line, stadia lines, horizontal line and micrometer setting V. The level vial has a sensitivity of 12″ per 2 mm. The manufacturers claim an accuracy of ± 0.02 inch/mile or ± 0.3 mm/km of single levelling.

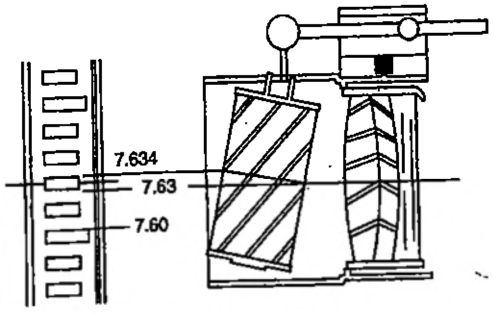


FIG. 17.5

For taking accurate staff reading, the micrometer screw is turned till the particular staff division line is brought in coincidence with the V of the reticule. This is accomplished by a parallel plate micrometer (Fig. 17.5) which measures the interval between the reticule line and the nearest division on the staff to an accuracy of 0.001 ft.

The device consists of parallel plate of glass which may be fitted to displace the rays of light entering the objective. The displacement is controlled by a micrometer screw (6) calibrated to give directly the amount of the interval.

17.5. ENGINEER'S PRECISE LEVEL (FENNEL)

Fig. 17.6 shows the photograph of Fennel's A 0026 precise Engineer's level with optical micrometer. It is equipped with a tilting screw and a horizontal glass circle. The coincidence of the bubble ends can be directly seen in the field of view of telescope. This assures exact centering of the bubble, when the rod is read. Fig. 17.7 (*a*) shows the telescope field of view when spirit level is not horizontal. Fig. 17.7 (*b*) shows the telescope field of view when the spirit level is horizontal. The sensitivity of tubular spirit level is 2″ per 2 mm. The optical micrometer is used for fine reading of staff. Fig. 17.7 (*c*) shows the field of view of optical micrometer for fine reading of the staff. The telescope has magnification of 32 dia. The horizontal glass circle—reading 10 minutes, estimation 1 minute—renders the instrument excellent for levelling tacheometry when used in conjuction with the Reichenback stadia hairs.

17.6. FENNEL'S FINE PRECISION LEVEL

Fig. 17.8 shows Fennel's 0036 fine precision level with optical micrometer. The length of the telescope, including optical micrometer is 15 inches, with $2\frac{1}{4}$ inch aperture of object glass and a magnifying power of $50\times$. The sensitivity of circular spirit level is 6′ while that of the tubular spirit level 10″ per 2 mm.

The bubble ends of the main spirit level are kinematically supported in the field of view, where they are read in coincidence (Fig. 17.8). A scale, arranged in the field of view, provides the reading of differences of variation of the bubble. The instrument is provided with wooden precision rod as well as invar tape rod, 3 m long with half centimetre graduated. Centimetre reading is directly read in the field of view of the telescope. Fine reading of the staff is read through separate microscope mounted adjacent the eyepiece. A scale permits direct readings of 1/10 of the rod interval and estimations of 1/100. Thus, in Fig. 17.9, the rod reading is = 244 + 0.395 = 244.395. A mean error of ± 0.3 to ± 0.5 mm per kilometre of double levelling is well obtainable with this instrument, if all precautions of precise levelling work are complied with.

17.7. PRECISE LEVELLING STAFF

For levelling of the highest precision, an Invar rod is used. Fig. 17.10 shows invar rod by M/s. Wild Heerbrugg Ltd. An invar band bearing the graduation is fitted to a wooden staff, tightly fastened at the lower end and by a spring at the upper end. Thus any extension of the staff has no influence on the invar band. The thermal expansion of the invar is practically nil. The graduations are of 1 cm. Two graduations mutually are displaced against each other to afford a check against gross errors. The length of graduations is 3 m. For measuring, the rod is always set up on an iron base plate. Detachable stays are provided for accurately and securely mounting the invar levelling staff. Once the rod is approximately vertical, the ends of stays are clamped tight. By means of the slow motion screw, the spherical level of the rod can be centred accurately.

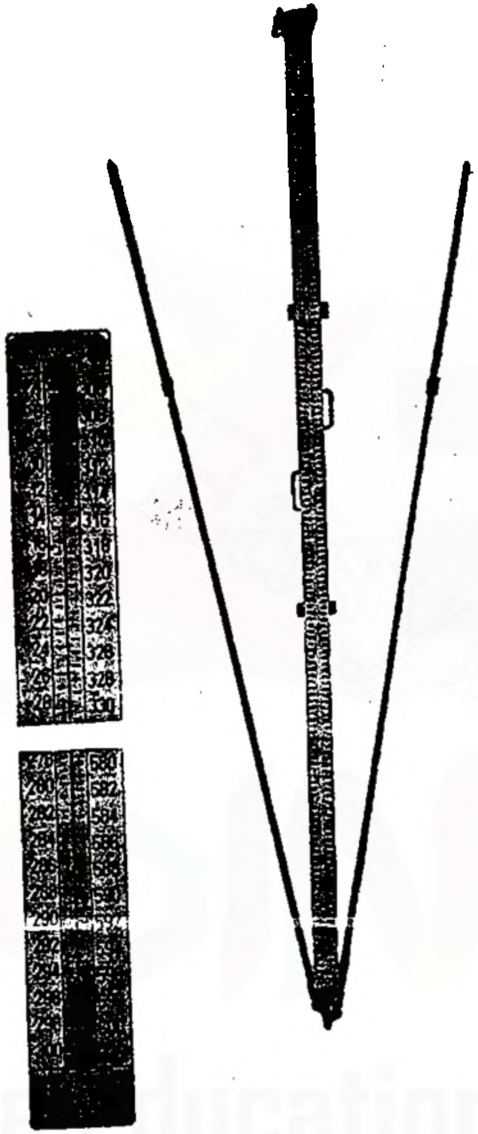


FIG. 17.10 INVAR PRECISION LEVELLING ROD (BY COURTESY OF M/S. WILD HEERBRUGG LTD.)

17.8. FIELD PROCEDURE FOR PRECISE LEVELLING

Two rod men are used ; they may be designated rod man *A* and rod man *B*. The rod *A* is called the B.M. rod. The rod *A* is held on the benchmark and the *B* rod on the turning point. After setting the level, micrometer is set at the *reversing point*. The longitudinal bubble is brought to its centre by micrometer screw before taking any reading. The first reading is taken on *A* rod and the second reading is taken on *B* rod placed at the turning point such that the backsight and foresight

distances are approximately equal. For each reading, all the three wires are read. When the instrument is moved, the *B* rod is left at the first turning point and the *A* rod is moved to the second turning point. At the second set up the level man reads rod *A* (foresight) first and then the rod *B* (backsight). When the instrument is moved again, the *A* rod is held where it is and the *B* rod is moved. At third set up of the level, the level man reads rod *A* (backsight) first and the rod *B* (foresight) next. *Thus, at alternate set up the foresight is read before the backsight and at every set up the A rod is read first and B rod next. The procedure neutralizes the effect of changing conditions like sinking of the level or changing refraction.* If it should happen that the *B* rod normally comes to the B.M. at the end of a section of levels, it is not used. Instead, the *A* rod is moved to the B.M. Thus, both sights at this instrument position are taken on the *A* rod. *This procedure eliminates any difference in index correction of the rod.*

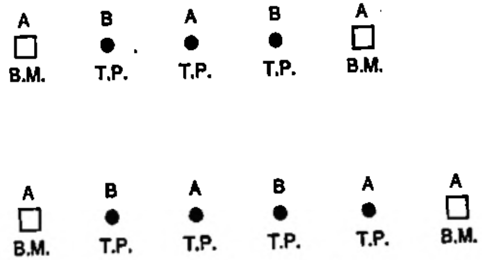


FIG. 17.11

In order to eliminate serious systematic errors due to the variations in temperature and refraction, each *section* is to be checked by a forward and a backward running—the forward running may be in the morning and backward in the afternoon. The difference in elevation obtained by these two runs should be checked within the limits of accuracy desired. The length of a section should not be more than 1200 metres.

If the work proceeds without interruption and no sudden change in temperature occurs, it is sufficient to record the two rod temperatures at the beginning and end of the section. The level should be protected from the sun. A rod level must be used to plumb the rod at all readings.

17.9. FIELD NOTES

The arrangement of level notes is almost similar to that of ordinary levelling, except that all the three cross-wire readings are taken and recorded. A line is drawn after three readings and average is found. This average gives the backsight or foresight reading at the point. The intervals between the top and central-wire and between bottom and central-wire readings are computed. The difference between these two interval readings should not be more than 0.005 ft or another set of readings must be taken. The difference between the top and bottom-wire readings (or the sum of the above calculated intervals) is a *measure* of the distance from the level to the rod and is called the *distance reading*. Starting with the first backsight, the distance reading of each successive backsight is added. Similarly, the distance reading of each successive foresight is also added. Thus, at any turning point, the sum thus formed gives the *total* of distances of backsights or foresights, as the case may be. The sum of total backsight distances must approximately be equal to the sum of total foresight distance at any turning point. The table below shows a page from precise level book.

A PAGE FROM PRECISE LEVEL BOOK

Station	B.S.	H.I.	F.S.	Elv.	Distance B.S.	Distance F.S.	Remarks
B.M.	2.623						
	3.346				0.723		
	4.070				0.724		
	3.346	528.125'		524.779'	1.447		
T.P. 1	3.825		3.986				
	4.506		4.706		0.681	0.720	
	5.189		5.428		0.683	0.722	
	4.507	527.925'	4.707	523.418'	2.811	1.444	
T.P. 2	4.685		3.628				
	5.610		4.280		0.925	0.652	
	6.534		4.930		0.924	0.650	
	5.610	529.256'	4.279	523.646'	4.660	2.746	
B.M.			4.960				
			5.890			0.930	
			6.822			0.932	
			5.891	523.365'		4.608	
Check	13.463		14.877	524.779			
			13.463	523.365			
		Fall	1.414	1.414	Fall		

17.10. DAILY ADJUSTMENTS OF PRECISE LEVEL

The adjustments of a precise level should be *tested* daily. If the adjustments are out by permissible amount, corrections are applied to the observations of the day's work. If, however, the adjustments are out by appreciable amount, they are adjusted. The following adjustments are made :

(*i*) Adjustment for circular bubble,
(*ii*) Adjustment for prism mirror,
(*iii*) Adjustment for the size of the bubble tube,
(*iv*) Adjustment for the line sight, and
(*v*) Adjustment for the reversing point.

(*i*) Adjustment for circular bubble

Centre the circular bubble by means of foot screws. Reverse the telescope. If the bubble moves from the centre, bring it half way back by means of the adjusting screws.

(*ii*) Adjustment of the prism mirror

With the right eye in position at the eyepiece, sight the prism mirror with the left eye. Swing the mirror until the bubble appears to be evenly situated to the centre line.

(*iii*) Adjustment for the size of the bubble tube

This adjustment can be made only if the level vial has an adjustable air chamber. If it has air chamber, the length of the bubble can be changed by tilting the chamber. Thus, to enlarge the bubble, tilt the eyepiece and upward and to decrease it, turn the eyepiece end downward.

(*iv*) Adjustment for the line of sight

The test of the parallelism of the line of sight and the axis of the bubble tube is of prime importance and shall be made daily. It may not be necessary to make the adjustment daily. However, the error is determined and correction is applied to the observed readings.

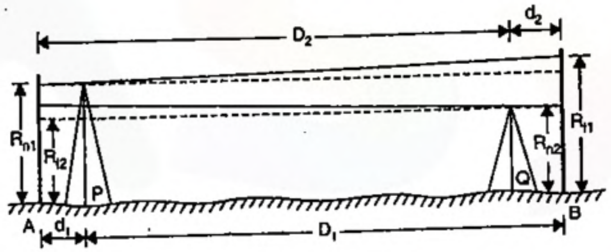


FIG. 17.12

To test the adjustment, two points A and B are selected about 120 m apart. The level is first set at P, near to A, at a distance d_1 from A and D_1 from B. Let the reading obtained at A be R_{n1} and that at B be R_{f1} , the suffix n and f being used to denote the readings on near and far points. The instrument is then moved to a point Q, near to B, at distance d_2 from B and D_2 from A. Let the reading obtained at A be R_{f2} and at B be R_{n2}. Let c = slope of the line of sight = tan α .

When the instrument is at P

True difference in elevation between A and $B = (R_{f1} - cD_1) - (R_{n1} - cd_1)$ ...(1)

When the instrument is at Q

The difference in elevation between A and $B = (R_{n2} - cd_2) - (R_{f2} - cD_2)$ ...(2)

Equating these two and solving for c, we get

$$c = \frac{(R_{n1} + R_{n2}) - (R_{f1} + R_{f2})}{(D_1 + D_2) - (d_1 + d_2)} = \frac{\text{Sum of near rod readings} - \text{Sum of far rod readings}}{\text{Sum of far distances} - \text{Sum of near distances}}$$

Knowing c, the correction to any rod reading can be calculated.

The line of sight will be inclined downwards if c has plus sign and will be inclined upwards if c has minus sign. If the value of c comes out to be more than 0.00005 (*i.e.* 0.005 m in 100 m), adjustments should be made by calculating the correction for a staff kept at 90 m distance from the instrument.

(*v*) **Adjustment of the reversing point**

The reversing point is a particular reading on the micrometer screw at which the bubble will remain central after reversal, when the vertical axis of the level is truly vertical. To find the reversing point, the bubble tube is centred exactly and the micrometer reading is noted. The telescope is then reversed, the bubble again centred and the micrometer reading is noted. The reversing point is then half-way between the two micrometer readings. The adjustment is not *essential* but is merely *necessary* for quick centring of the bubble at all times. Whenever the instrument is being levelled, the micrometer screw should be set at the reversion point.

18

Permanent Adjustments of Theodolite

18.1. GENERAL

The *fundamental lines* of a transit are as follows :

(1) The vertical axis

(2) The horizontal axis

(3) The line of collimation (or line of sight)

(4) Axis of plate level

(5) Axis of altitude level

(6) Axis of the striding level, if provided.

The following *desired relations* should exist between these lines :

(1) The axis of the plate level must lie in a plane perpendicular to the vertical axis.

If this condition exists, the vertical axis will be truly vertical when the bubble is in the centre of its run.

(2) The line of collimation must be perpendicular to the horizontal axis at its intersection with the vertical axis. Also, if the telescope is external focusing type, the optical axis, the axis of the objective slide and the line of collimation must coincide.

If this condition exists, the line of sight will generate a vertical plane when the telescope is rotated about the horizontal axis.

(3) The horizontal axis must be perpendicular to the vertical axis.

If this condition exists, the line of sight will generate a vertical plane when the telescope is plunged.

(4) The axis of the altitude level (or telescope level) must be parallel to the line of collimation.

If the condition exists, the vertical angles will be free from index error due to lack of parallelism.

(5) The vertical circle vernier must read zero when the line of collimation is horizontal.

If this condition exists, the vertical angles will be free from index error due to displacement of the vernier.

(6) The axis of the striding level (if provided) must be parallel to the horizontal axis.

Due to collimation error each backsight staff reading is too great by an amount (80 tan 60″) metres. Also each change point *F.S.* reading is too great by an amount (30 tan 60″) metre. Taking both errors together, it is as if *F.S.* readings were correct and *B.S.* too great by amount (50 tan 60″) metres.

As there are four set-ups, the total *B.S.* reading are great by an amount $4 \times 50 \tan 60''$ $= 200 \times 0.0002909 = 0.05818 \approx 0.058$ metres. Now greater the *B.S.* readings, higher will be the *H.I.* and, therefore, greater will be reduced levels calculated. The actual level of the *T.B.M.* will therefore, be $= 54.065 - 0.058 =$ **54.007 m.**

9.12. CURVATURE AND REFRACTION

From the definition of a level surface and a horizontal line it is evident that a horizontal line departs from a level surface because of the curvature of the earth. Again, in the long sights, the horizontal line of sight does not remain straight but it slightly bends downwards having concavity towards earth surface due to refraction.

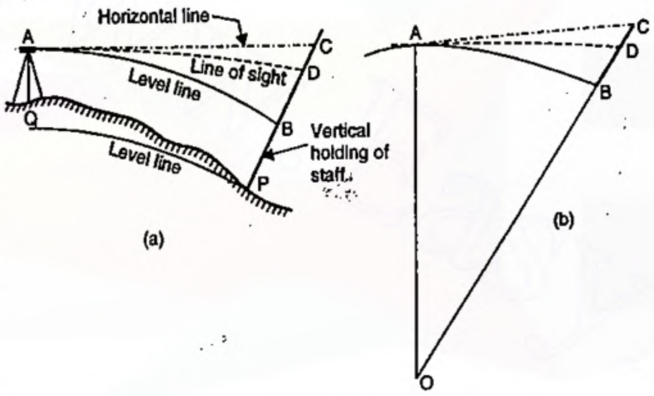


FIG. 9.38. CURVATURE AND REFRACTION.

In Fig. 9.38 (*a*), *AC* is the horizontal line which deflects upwards from the level line *AB* by an amount *BC*. *AD* is the actual line of sight.

Curvature. *BC* is the departure from the level line. Actually the staff reading should have been taken at *B* where the level line cuts the staff, but since the level provides only the horizontal line of sight (in the absence of refraction), the staff reading is taken at the point *C*. Thus, the apparent staff reading is *more* and, therefore, the object appears to be *lower* than it really is. *The correction for curvature is, therefore, negative as applied to the staff reading,* its numerical value being equal to the amount *BC*. In order to find the value *BC*, we have, from Fig. 9.38 (*b*).

$$OC^2 = OA^2 + AC^2, \ \angle CAO \text{ being } 90°$$

Let $BC = C_c =$ correction for curvature

$AB = d =$ horizontal distance between *A* and *B*

$AO = R =$ radius of earth in the same unit as that of *d*

$$\therefore \quad (R + C_c)^2 = R^2 + d^2$$

or $$R^2 + 2RC_c + C_c^2 = R^2 + d^2$$

$$\therefore \quad C_c(2R + C_c) = d^2$$

or $$C_c = \frac{d^2}{2R + C_c} \simeq \frac{d^2}{2R}, \text{ (Neglecting } C_c \text{ in comparsion to } 2R)$$

That is, to find the curvature correction, divide the square of the length of sight by earth's diameter. Both *d* and *R* may be taken in the same units, when the answers will also be in terms of that unit. The radius of the earth can be taken equal to 6370 km. If *d* is to be in km, and $R = 6370$ km, $\boxed{C_c = 0.07849\, d^2 \text{ metres.}}$ In the above expression, *d* is to be substituted in km, while C_c will be in metres.

Refraction : The effect of refraction is the same as if the line of sight was curved downward, or concave towards the earth's surface and hence the rod reading is decreased. Therefore, the effect of refraction is to make the objects appear higher than they really are. *The correction, as applied to staff readings, is positive.* The refraction curve is irregular because of varying atmospheric conditions, but for average conditions it is assumed to have a diameter about seven times that of the earth.

The correction of refraction, C_r is therefore, given by

$$\boxed{C_r = \frac{1}{7}\,\frac{d^2}{2R}\,(+\text{ve}) = 0.01121\, d^2 \text{ metres,}} \text{ when } d \text{ is in km.}$$

The combined correction due to curvature and refraction will be given by

$$\boxed{\begin{aligned} C &= \frac{d^2}{2R} - \frac{1}{7}\,\frac{d^2}{2R} = \frac{6}{7}\,\frac{d^2}{2R} \\ &= 0.06728\, d^2 \text{ metres,} \end{aligned}} \text{ (subtractive) } d \text{ being in km.}$$

The corresponding values of the corrections in English units are :

$$\left.\begin{aligned} C_c &= \tfrac{2}{3}\, d^2 = 0.667\, d^2 \text{ feet} \\ C_r &= \tfrac{2}{21}\, d^2 = 0.095\, d^2 \text{ feet} \\ C &= \tfrac{4}{7}\, d^2 = 0.572\, d^2 \text{ feet} \end{aligned}\right] \begin{array}{l} d \text{ is in miles and} \\ \text{radius of earth} = 3958 \text{ miles.} \end{array}$$

Distance to the visible horizon

In Fig. 9.39, let *P* be the point of observation, its height being equal to *C* and let *A* be the point on the *horizon i.e.*, a point where the tangent from *P* meets the level line. If *d* is the distance to visible horizon, it is given by

$$\boxed{\begin{aligned} d &= \sqrt{\frac{C}{0.06728}} \text{ km} \\ &= 3.8553\sqrt{C} \text{ km, } C \end{aligned}} \text{ being in metres.}$$

(Taking both curvature and refraction into account).

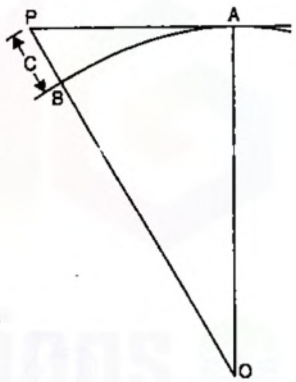


FIG. 9.39.

Example 9.8. *Find the correction for curvature and for refraction for a distance of (a) 1200 metres (b) 2.48 km.*

Solution.

(*a*) Correction for curvature $= 0.07849\, d^2$ metres (where *d* is in km)

$= 0.07849\,(1.2)^2 = \mathbf{0.113}$ **m**

Correction for refraction $= \frac{1}{7}\, C_c = \mathbf{0.016}$ **m** (*b*) Correction for curvature $= 0.07849\ (2.48)^2 = \mathbf{0.483}$ m

Correction for refraction $= \frac{1}{7} C_c = \mathbf{0.069}$ m.

Example 9.9. *Find the combined correction for curvature and refraction for distance of (a) 3400 metres (b) 1.29 km.*

Solution.

(*a*) Combined correction for curvature and refraction

$$= 0.06728\ d^2 \text{ m} = 0.06728\ (3.40)^2 = \mathbf{0.778} \text{ m.}$$

(*b*) Combined correction $= 0.06728\ (1.29)^2 = \mathbf{0.112}$ m.

Example 9.10. *In order to find the difference in elevation between two points P and Q, a level was set upon the line PQ, 60 metres from P and 1280 metres from Q. The readings obtained on staff kept at P and Q were respectively 0.545 metre and 3.920 m. Find the true difference in elevation between P and Q.*

Solution.

Since the distance of P from instrument is small, the correction for curvature etc. is negligible.

Combined correction for Q $= 0.06728\ (1.280)^2 = 0.110$ m (Subtractive)

$\therefore$ Correct staff reading at Q $= 3.920 - 0.110 = 3.810$ m

$\therefore$ Difference in elevation between P and $Q = 3.810 - 0.545 = \mathbf{3.265}$ m, Q being lower.

Example 9.11. *A light-house is visible just above the horizon at a certain station at the sea level. The distance between the station and the light-house is 50 km. Find the height of the light-house.* (combined correction)

Solution.

The height of the light-house is given by

$$C = 0.06728\ d^2 \text{ metres} = 0.06728\ (50)^2 \text{ metres} = \mathbf{168.20} \text{ m}$$

Example 9.12. *An observer standing on the deck of a ship just sees a light-house. The top of the light-house is 42 metres above the sea level and the height of the observer's eye is 6 metres above the sea level. Find the distance of the observer from the light-house.*

Solution. (Fig. 9.40)

Let A be the position of the top of light-house and B be the position of observer's eye. Let AB be tangential to water surface at O.

The distances d_1 and d_2 are given by

$$d_1 = 3.8553 \sqrt{C_1} \text{ km}$$

$$= 3.8553 \sqrt{42} = 24.985 \text{ km}$$

and $$d_2 = 3.8553 \sqrt{6} = 9.444 \text{ km}$$

$\therefore$ Distance between A and $B = d_1 + d_2$

$= 24.985 + 9.444 = \mathbf{34.429}$ km.

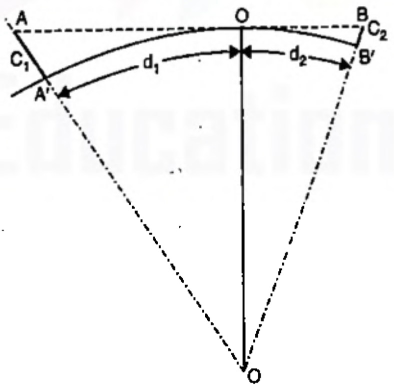


FIG. 9.40.

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Example 9.13. *The observation ray between two triangulation stations A and B just grazes the sea. If the heights of A and B are 9,000 metres and 3,000 metres respectively, determine approximately the distance AB (Diameter of earth 12,880 km).*

Solution.

In Fig. 9.40, let A and B be the two triangulation stations and let O be the point of tangency on the horizon.

Let $A'A = C_1 = 9000$ metres $= 9$ km

$B'B = C_2 = 3000$ metres $= 3$ km

$d \approx 3.8553 \sqrt{9000}$

The distance d_1 is given by $C_1 = \dfrac{d_1^2}{2R}$

or $d_1 = \sqrt{2\,RC_1}$ in which d_1, R and C_1 are in same units

$\therefore$ $d_1 = \sqrt{2 \times 6440 \times 9.0} = 340.48$ km

Similarly $d_2 = \sqrt{2RC_2} = \sqrt{2 \times 6440 \times 3.0} = 196.58$ km

$\therefore$ Distance $AB = d_1 + d_2 = 340.48 + 196.58 = \mathbf{537.06}$ **km.**

Example 9.14. *Two pegs A and B are 150 metres apart. A level was set up in the line AB produced and sights were taken to a staff held in turn on the pegs, the reading being 1.962 (A) and 1.276(B), after the bubble has been carefully brought to the centre of its run in each case. The reduced level of the tops of the pegs A and B are known to be 120.684 and 121.324 m respectively.*

Determine (a) the angular error of the collimation line in seconds, and (b) the length of sight for which the error due to curvature and refraction would be the same as collimation error. Assume the radius of the earth to be 6370 km.

Solution.

Observed difference in elevation between A and $B = 1.962 - 1.276 = 0.686$ m (A being lower)

The difference in elevation $= 121.324 - 120.684 = 0.640$ m, A being lower.

Hence, from the observations, A seems to be lower by an additional amount $= 0.686 - 0.640$
$= 0.046$ m.

Since B is nearer to the instruments than A, it is clear that the line of sight is inclined upwards by an amount 0.046 m in a length of 150 m.

If α is the angular inclination (upwards) of the line of sight with horizontal,

$$\tan \alpha = \frac{0.046}{150} = 0.0003067$$

We know that $\tan 60'' = 0.0002909$

$\therefore$ $$\alpha = \frac{3067 \times 60}{2909 \times 60} \text{ minutes} = \mathbf{1'\ 3''} \text{ (upwards).}$$

For the second part of the problem, let the required line of sight be L km. The combined correction for curvature and refraction would be $\frac{6}{7} \frac{L^2}{2R}$ (negative). The correction for collimation error in a length L will be $L \tan \alpha$. Equating the two,

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$$\frac{6}{7}\,\frac{L^2}{2R} = L \tan\alpha = L \;\; (0.0003067)$$

$$\therefore \qquad L = \frac{0.0003067 \times 7 \times 2}{6} \times 6370 = \mathbf{4.557} \;\; \textbf{km.}$$

9.13. RECIPROCAL LEVELLING

When it is necessary to carry levelling across a river, ravine or any obstacle requiring a long sight between two points so situated that no place for the level can be found from which the lengths of foresight and backsight will be even approximately equal, special method *i.e., reciprocal levelling* must be used to obtain accuracy and to eliminate the following: (1) error in instrument adjustment ; (2) combined effect of earth's curvature and the refraction of the atmosphere, and (3) variations in the average refraction.

Let A and B be the points and observations be made with a level, the line of sight of which is inclined upwards when the bubble is in the centre of its run. The level is set at a point near A and staff readings are taken on A and B with the bubble in the centre of its run. Since B.M. A is very near to instrument, no error due to curvature, refraction and collimation will be introduced in the staff readings at A, but there will be an error e in the staff reading on B. The level is then shifted to the other bank, on a point very near B.M. B, and the readings are taken on staff held at B and A. Since B is very near, there will be no error due to the three factors in reading the staff, but the staff reading on A will have an error e. Let h_a and h_b be the corresponding

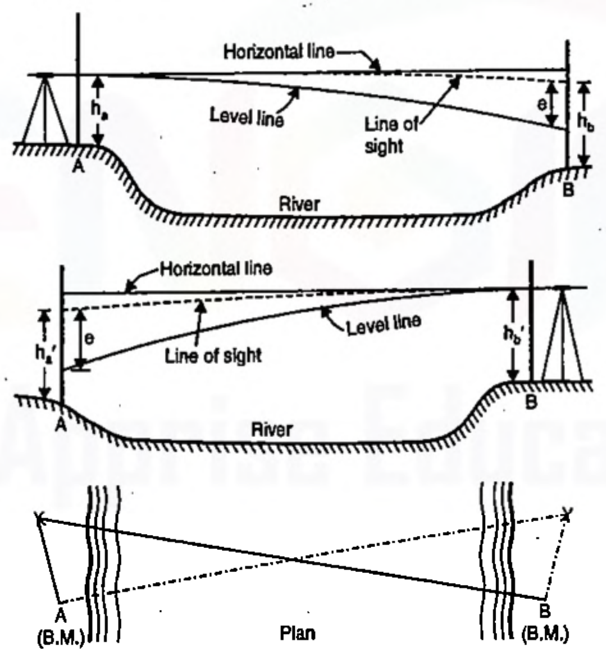

FIG. 9.41. RECIPROCAL LEVELLING.

staff readings on A and B for the first set of the level and h_a' and h_b' be the readings for the second set.

From Fig. 9.41, it is evident that for the first set of the level, the correct staff readings will be

$$\text{On } A : h_a \;; \qquad \text{On } B : h_b - e$$

$\therefore$ True difference in elevation $= H = h_a - (h_b - e)$

Similarly for second set, the correct staff reading will be :

$$\text{On } A : h_a' - e \;; \qquad \text{On } B : h_b'$$

$\therefore$ True difference in elevation $= H = (h_a' - e) - h_b'$

Taking the average of the two true differences in elevations, we get

$$2H = [h_a - (h_b - e) + (h_a' - e) - h_b'] = (h_a - h_b) + (h_a' - h_b')$$

$$\therefore \qquad H = \tfrac{1}{2}[(h_a - h_b) + (h_a' - h_b')] \;.$$

The true difference in elevation, therefore, is equal to the mean of the two appearent differences in elevations, obtained by reciprocal observations.

Example 9.15. *The following notes refer to reciprocal levels taken with one level:*

Inst. at	*Staff readings on*		*Remarks*
	P	*Q*	
P	*1.824*	*2.748*	*Distance between P and Q=1010 m*
Q	*0.928*	*1.606*	*R.L. of P=126.386.*

Find (a) true R.L. of Q, (b) the combined correction for curvature and refraction, and (c) the angular error in the collimation adjustment of the instrument.

What will be the difference in answers of (a) and (c) if observed staff readings were 2.748 on P and 1.824 on Q, the instrument being at P ; and 1.606 on P and 0.928 on Q, the instrument being at Q.

Solution.

(*a*) When the observations are taken from P, the apparent difference in elevation between P and $Q = 2.748 - 1.824 = 0.924$ m, P being higher

When the observations are taken from Q, the apparent difference in elevation between P and $Q = 1.606 - 0.928 = 0.678$, P being higher.

Hence, the true difference in elevation

$$= \frac{0.924 + 0.678}{2} = 0.801 \;\text{m}, \; P \text{ being higher}$$

and true elevation of $Q = 126.386 - 0.801 = \mathbf{125.585}$ m.

(*b*) Combined correction for curvature and refraction

$$= 0.06728\, d^2 = 0.06728\,(1.010)^2 = \mathbf{0.069} \;\text{m}$$

(Q appears to be lower further by 0.069 m due to this)

(*c*) When the level was at P, the apparent difference in elevation $= 0.924$ m.

The difference in elevation $= 0.801$ m